A general statistical mechanical approach for deriving parametrizations

Lucarini Valerio [1,2]

1 : Klimacampus, Institute of Meteorology, University of Hamburg (Klimacampus)
http://www.klimacampus.de/
2 : Department of Mathematics and Statistics, University of Reading (UOR)
The University of Reading, Whiteknights, PO Box 217, READING, Berkshire, RG6 6AH, United Kingdom
http://www.rdg.ac.uk/

We consider the problem of deriving approximate autonomous dynamics for a number of variables of a dynamical system, which are weakly coupled to the remaining variables. Our findings have relevance for the construction of parametrizations of unresolved processes in many non-equilibrium systems, and most notably in geophysical fluid dynamics. We first propose a systematic way to construct a surrogate dynamics, such that the expectation value of any observable agrees, up to second order in the coupling strength, to its expectation evaluated on the full dynamics. By direct calculation, we find that, at first order, the coupling can be surrogated by adding a deterministic perturbation to the autonomous dynamics of the system of interest. At second order, there are two separate and very different contributions. One is a term taking into account the second order contributions of the fluctuations in the coupling, which can be parametrized as a stochastic forcing with given spectral properties. The other one is a memory term, coupling the system of interest to its previous history, through the correlations of the second system. If these correlations are known, this effect can be implemented as a perturbation with memory on the single system. Furthermore, we show that such surrogate dynamics agrees up to second order to an expansion of the Mori-Zwanzig projected dynamics. This implies that the parametrizations of unresolved processes suited for prediction and for the representation of long term statistical properties are closely related, if one takes into account, in addition to the widely adopted stochastic forcing, the often neglected memory effects. We emphasize that our results do not rely on assuming a time scale separation, and, if such a separation exists, can be used equally well to study the statistics of the slow as well as that of the fast variables. The results bear relevance also in the context of the applicability of the fluctuation-dissipation relation for geophysical fluid dynamical systems. [1,2].

[1] J. Wouters and V. Lucarini, Disentangling multi-level systems: averaging, correlations and memory, J. Stat. Mech. P03003 doi:10.1088/1742-5468/2012/03/P03003 (2012)
[2] J. Wouters and V. Lucarini, Multi-level Dynamical Systems: Connecting the Ruelle Response Theory and the Mori-Zwanzig Approach, J. Stat. Phys., doi: 10.1007/s10955-013-0726-8 (2013)