Limits of balance in rotating stratified flows

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A major unresolved question is: «under what conditions does a rotating, stably-stratified flow exhibit a forward energy cascade at small scales?» Put a different way, «what conditions lead to the breakdown of (geostrophic, hydrostatic or higher-order) balance?»
This question has been the subject of many papers, with no consensus. The key issue is: can an initially balanced flow spontaneously generate a significant amount of inertia?gravity waves? Moreover, do these waves ? via nonlinear interactions ? exhibit a forward energy cascade, possibly explaining the transition from a k ?3 intermediate to large-scale spectrum to a k ?5/3 small-scale spectrum claimed to exist in the Earth's atmosphere (Gage & Nastrom, 1985)?
A number of researchers claim that ageostrophic motions, developing from a loss of balance in an initially balanced flow, lead to a downscale energy flux ? a direct internal route for a flow to cascade to small scales, independent of boundary influences (Bartello 1985, Molemaker et al. 2005, Waite & Bartello 2006, Bartello 2010, Nadiga & Straub 2010, Nadiga 2013). These studies have considered the prototype non-hydrostatic Boussinesq equations, the simplest fluid dynamical model
containing both balanced vortical motions and unbalanced inertia?gravity waves, in a simple triply-periodic geometry (no boundaries). Other studies considering the same equations, however, have reached strongly contrasting conclusions (Dritschel & Viúdez 2003, 2007; Viúdez & Dritschel 2003, 2004, 2006; McKiver & Dritschel 2008). Instead of finding a direct energy cascade of ageostrophic motions to small scales, these studies demonstrated an extraordinarily weak generation of inertia?gravity waves and a near-complete dominance of balanced, vortical motions.
What could explain the differences? First, the latter studies better prepared the initial conditions to reduce the initial imbalance. Second, the latter studies used explicit potential vorticity conservation and a variable transformation to separate balanced and unbalanced motions at leading order (thermal-wind balance). This demonstrably reduces the false, numerical generation of inertia?gravity waves. Third, the latter studies used an anisotropic grid, with the vertical to horizontal grid box ratio scaling with the ratio f /N (Coriolis to buoyancy frequency), whereas the former studies used an isotropic grid.
This talk will discuss these and other differences, and propose a way forward to distinguish regimes of motion dominated by balance (as observed in the latter studies) from those exhibiting a strong degree of imbalance.

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